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TRANSIENT RADIATIVE COOLING OF A SEMI-INFINITE SOLID WITH PARALLEL-WALLED CAVITIES

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Abstract—A semi-infinite solid of blackbody material is separated into slabs by periodically spaced in-infinitely deep cavities bounded by plane parallel walls. The solid material is brought to uniform initial temperature and subsequently is allowed to cool by conduction in the interior and by radiation at the surfaces. The temperature history of the solid with cavities was calculated numerically. Similar calculations were performed to obtain the rate of cooling of the same material without cavities. Comparison is made between the apparent brightness temperatures measured at a great distance by a thermal radiation detector with a resolving power insufficient to discriminate between a smooth surface and one with cavities. It is shown that over a wide range of aspect angles the solid with cavities may have a higher apparent temperature even when the top surfaces of the slabs are cooler than the surface of a homogeneous solid. Phenomena such as the one described may be useful in the interpretation of measurements of infrared radiation from the lunar surface.

NOMENCLATURE

- c, specific heat;
- d, dimensionless depth at which temperature is sensibly constant for $t \leqslant 10$;
- H, average dimensionless rate of heat flow per unit area at site of thermal detector, per unit area of surface $y = 0$;
- K, thermal conductivity;
- r, dimensionless distance to detector;
- s, dimensionless width of slab;
- t, dimensionless time;
- T, dimensionless temperature;
- T_b, apparent brightness temperature, defined by equations (6) and (7);
- T_i, initial temperature;
- w, dimensionless width of cavity;
- x, y, dimensionless coordinates in directions parallel to top and side surfaces of solid (cavity wall), respectively.

Greek symbols

- ρ, mass density;
- σ, Stefan–Boltzmann constant;
- ϕ, angle between normal to top surface of solid and principal axis of thermal detector response pattern.

INTRODUCTION

THERMAL radiation from cavities and enclosures in a semi-infinite solid has long been a problem of practical importance. In recent years several significant developments have been reported in the literature (see, for example, references [1–4]). Reviews of cavity radiation studies have been published by several workers, including Williams [5] and Viskanta and Grosh [6]. An extensive bibliography is to be found in the latter survey.

Nearly all discussions of thermal radiation from cavities assume that the solid material bounding the enclosure emits and reflects radiation in a perfectly diffuse manner; exceptions are studies by DeVos and Edwards, as described by Williams [5]. Furthermore, it is usually assumed that a steady state prevails; either the walls of the cavities are maintained at a known constant temperature or a steady heat flux is specified. So far as the author is aware, time-dependent problems in cavity radiation theory have not yet been considered. The purpose of this paper is to discuss the transient temperature of a semi-infinite solid, containing cavities, which radiates into a zero temperature medium. Since the principal objective of this study was to describe the general characteristics of the phenomenon, the analytic complexity was minimized by considering only periodically spaced cavities bounded by plane parallel walls and by assuming the solid to be composed of perfectly absorbing material.

The solid cools by both radiation and conduction and its temperature history was calculated numerically, assuming a uniform starting temperature. Of particular interest is the temperature variation along the top of the solid and along the walls of the cavities at various stages of cooling. Comparison was made with the transient surface temperature of a radiating solid without cavities. This latter problem has already been discussed by several workers, including Jaeger [7], Abarbanel [8], and Lick [9]. As expected, it was found that a solid with infinitely deep rectangular cavities eventually loses heat energy more rapidly than one with a homogeneous surface. On the other hand, the differential cooling rate depends both upon the extent s of solid material between the walls and upon the width w of the cavity itself. Moreover, the average wall temperature, over distances of one or two w from the edge, can remain considerably higher than the average temperature of the top surface over significant intervals of time. Hence, if such a configuration were viewed from above at a great distance with a thermal detector incapable of resolving the declivities, the apparent temperature would, for some period of time, exceed that of a solid without cavities.

Finally, it may be pointed out that the solution of problems of this nature may be of some value in interpreting anomalous transient thermal radiation phenomena associated with a surface containing declivities such as that of the Moon (see, for example, Saari and Shorthill [10]).*

* Subsequent to the submission of the present paper, a similar observation has been made by J. A. Bastin [13] who suggests that preferential lunar surface roughness, such as declivities on a centimeter scale, may account for thermal anomalies found by Saari and Shorthill in infrared eclipse observations.

STATEMENT OF THE PROBLEM

Consider a semi-infinite solid of perfectly absorbing material, separated into slabs of width s, bounded at the sides by plane parallel walls separated by distance w (see Fig. 1). The medium external to the solid is taken to be free space.

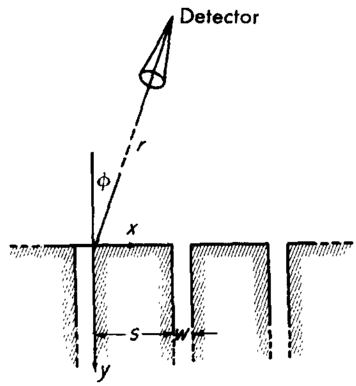

FIG. 1. Geometry of the problem of transient cooling of a semi-infinite solid with parallel-walled cavities.

The parameters which characterize the material are thermal conductivity K, mass density ρ, and specific heat c. For negative values of time the solid is maintained at a uniform temperature T_i by an appropriate distribution of heat sources. At time equal to zero the sources are suddenly turned off and the material cools by conduction in the interior and by radiation from the surfaces in accordance with the Stefan–Boltzmann Law.

Since a comparison is to be made between the transient temperatures of solids with and without cavities, it is convenient to nondimensionalize in a manner appropriate to the homogeneous semi-infinite solid problem. Thus, the temperature T is measured in units of T_i, distances in units of $K/(\sigma T_i^3)$, and time t in units of $K\rho c/(\sigma T_i^3)^2$, where σ is the Stefan–Boltzmann constant.

As a consequence of the assumption that the cavities are spaced periodically, the temperature history of only a single solid need be calculated. If s and w are now the nondimensional widths of the solids and the cavities, respectively, then the boundary value problem consists of solving the heat conduction equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\partial T}{\partial t} \quad (1)$$

in the domain $(0 \leqslant x \leqslant s; 0 \leqslant y \leqslant \infty)$ for $t \geqslant 0$, subject to the initial condition

$$T(x, y, 0) = 1. \quad (2)$$

Clearly, all of the thermal energy radiated from the top surface of the solid is permanently lost from the system, so that the radiation boundary condition at the top surface takes the form

$$\frac{\partial T}{\partial y} = T^4 \text{ at } y = 0. \quad (3)$$

However, when considering the heat flux from a point on a cavity wall, we note that only those photons which reach the space $y < 0$ are permanently lost; the remainder are absorbed by and reemitted from the opposite wall. Thus, the solids adjacent to the one under consideration provide radiative heat input at $x = 0$ and $x = s$. The appropriate conditions on the sides are easily derived when the assumption is made that the solid is composed of blackbody material (so that the reflectivity is zero and the emissivity is unity); the final results read

$$\frac{\partial T}{\partial x} = T^4 - \int_0^\infty T^4(0, y', t)\, K(y, y')\, \mathrm{d}y' \quad \text{at } x = 0 \quad (4a)$$

and

$$-\frac{\partial T}{\partial x} = T^4 - \int_0^\infty T^4(s, y', t)\, K(y, y')\, \mathrm{d}y' \quad \text{at } x = s, \quad (4b)$$

where

$$K(y, y') = \tfrac{1}{2}w^2[(y - y')^2 + w^2]^{-3/2}. \quad (4c)$$

Due to the complexity of the boundary conditions the problem was solved numerically. For the purpose of calculation a depth d was established* for the slab at which the departure of the temperature from unity was less than a prescribed amount over the maximum time interval of the calculation ($t = 10$). Assuming an appropriate d, equation (1), together with equations (2), (3) and (4) was solved numerically on an SRU 1107. The boundary conditions given above and the approximate condition $T = 1$ at $y = d$ were expressed as explicit difference equations, modified in a fashion described by Carslaw and Jaeger [11]. The equations for T at interior points were solved simultaneously with the boundary equations using the alternating-direction implicit method described by Peaceman and Rachford Jr. [12].

The transient temperature along the top of the solid and along the cavity walls is of special interest. For example, suppose the radiative heat flux from a semi-infinite blackbody solid containing cavities such as those described were measured from a great distance by a thermal detector which views a small solid angle. Consider the case in which the cavities are perpendicular to the plane determined by the principal axis of the detector response pattern and the normal projection of the axis on the top surface of the solid. Denote the angle between the normal to to the top surface of the solid and the principal axis of the detector response pattern by ϕ and denote by r the distance along the principal axis between the detector and the surface. At aspect angle ϕ the detector views a region R of the top surface of the solid. Suppose that the distance r is much greater than the largest dimension of R and further suppose that R is large enough to include a great many cavities. Subject to these restrictions, it is possible to define an average rate of heat flow H per unit area at the location of the detector, per unit area of the surface $y = 0$, measured in units of

* The effective depth was actually determined by a sequence of systematic trial calculations.

$(\sigma T_i^4)^3/(KT_i)^2$, by the approximate relation

$$H(\phi, t) \simeq (\pi r^2)^{-1}(s + w)^{-1} \{\cos \phi \int_0^s T^4(x, 0, t)\, dx + \sin \phi \int_0^{w \cot \phi} T^4(0, y, t)\, dy\}. \quad (5)$$

On the other hand, if the radiating medium were assumed to be a uniform blackbody solid, then an apparent brightness temperature T_b could be inferred from a measure of the heat flux H in accordance with

$$H(\phi, t) \simeq (\pi r^2)^{-1}\, T_b^4 \cos \phi. \quad (6)$$

Consequently, the brightness temperature T_b obtained by equating (5) and (6),

$$T_b = \{(s + w)^{-1} [\int_0^s T^4(x, 0, t)\, dx + \tan \phi \int_0^{w \cot \phi} T^4(0, y, t)\, dy]\}^{1/4}, \quad (7)$$

is the temperature which would be attributed to the medium from a measure of H if the presence of the cavities were not detected by some other means. Numerical studies described in the next section suggest that if cracks and declivities are, in fact, present in an apparently homogeneous solid, attempts to infer thermal properties from the transient behavior of H can be seriously in error. Such a situation could prevail in the interpretation of measurements of infrared radiation from the lunar surface.

NUMERICAL RESULTS

When the radiating semi-infinite solid problem is nondimensionalized in the manner described earlier, then a single calculation is sufficient to obtain the temperature variation in any material with constant thermal properties and a uniform starting temperature. Radiative cooling of a solid with the cavity configuration in this study is essentially a two parameter problem. Moreover, the numerical solution of the problem for each pair of parameters (w, s) is an undertaking of considerable magnitude. Therefore, only a few representative calculations were performed. The cases chosen for study correspond to the pairs $(w, s) = (1, 10), (1, 5), (2, 10)$ and $(2, 5)$. Although the numerical value of the temperature at any given point at time t depends upon both the cavity width w and the slab width s, certain general features of the thermal histories were similar in all four cases.

Figures 2 and 3 show the variation of the true temperature at several times along the top surface and along the cavity wall for the case $w = 2$ and $s = 5$. One rather interesting charac-

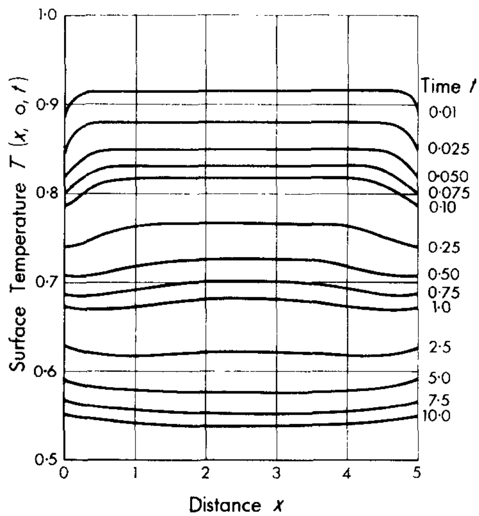

FIG. 2. Variation of temperature along the top surface of the solid at several times for the case $w = 2$, $s = 5$.

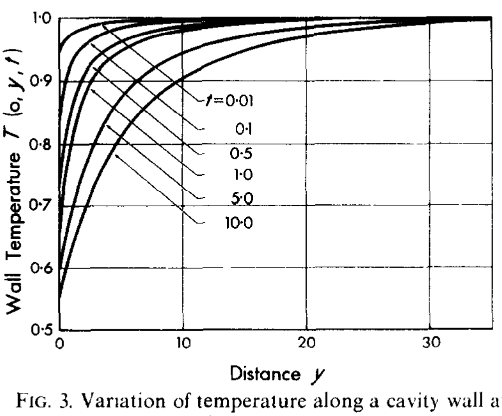

FIG. 3. Variation of temperature along a cavity wall at several times for the case $w = 2$, $s = 5$.

teristic of the cooling phenomenon is evident in Fig. 2 (this feature appeared in the other cases as well). The temperature gradient in the x-direction near the corners of the solid reverses sign as time increases. Moreover, as can be seen from Fig. 3, the magnitude of the temperature gradient in the y-direction always exceeds the gradient in the x-direction in the region of the corner. From these two observations we conclude that the corner regions always undergo a net loss of heat energy (as do the other areas of the surface) and that the loss by radiation from the top portion of the surface is dominant. The reversal of the sign of the temperature gradient in the x-direction comes about as the result of an interchange in the relative importance of heat energy loss by radiation from the surface and gain of heat energy as a consequence of incoming radiation from the opposite cavity wall. For t somewhat less than unity, the loss from outgoing radiation exceeds the gain from incoming radiation; at later times, when the temperature gradients are less steep, the opposite situation prevails.

Another feature of the phenomenon which appeared in all the configurations studied was the relatively small deviation of the temperature along the top surface from its average value at each instant of time (up to $t = 10$). In each case, moreover, the average temperature of the top surface of the solid at time t differed from that of a solid without cavities at the same time t by only a few per cent over the time interval of the calculation. During the initial stages of cooling, the top surface of the solid is generally at a higher temperature than that of a semi-infinite solid. At later times, however, the surface temperature of the solid with cavities falls below that of the uniform solid. For the configurations studied here, the cross-over usually occurred at some time between $t = 1$ and $t = 10$.

Figure 4 depicts the variation of apparent brightness temperature with time when the several surfaces are viewed from above at a great distance with a thermal detector with poor resolution characteristics. In each case the apparent

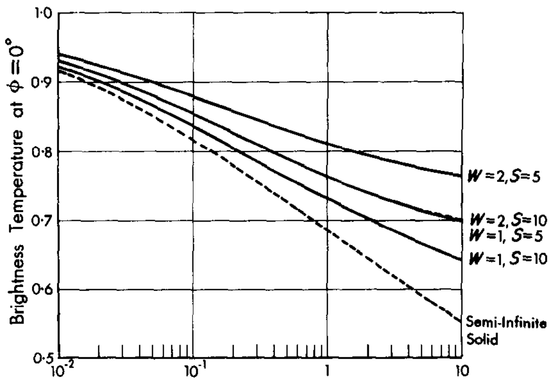

FIG. 4. Transient behavior of apparent brightness temperature at aspect angle $\phi = 0°$.

brightness temperature is somewhat higher than that of the uniform solid. Moreover, the disparity is significant over a wide range of aspect angles, as shown in Fig. 5 for the case $w = 1$ and $s = 5$.

In conclusion, the present results appear to indicate that the existence of cracks and declivities in an otherwise uniform solid can be of importance in determining the apparent surface temperature over periods of time long in comparison with the characteristic time of the homogeneous solid. Consequently, interpretive uncertainties may arise when declivities exist in a

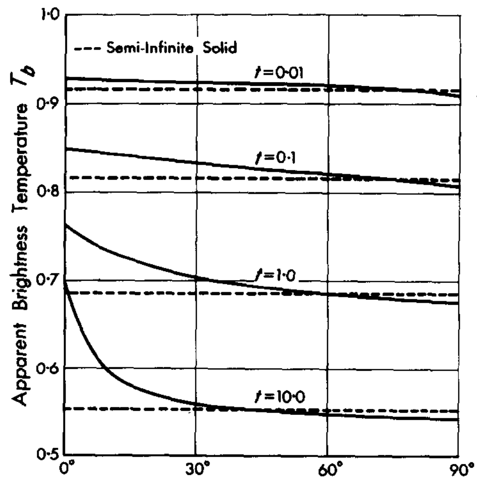

FIG. 5. Variation of apparent brightness temperature with aspect angle at several times for the case $w = 1, s = 5$.

surface, but their presence is not known to an observer using a thermal detector at great distance from the surface. If the observer assumes the surface to be homogeneous, then values of the thermal properties of the material which are inferred from the measured transient surface brightness temperature may differ considerably from the true values.

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Résumé—Un solide semi-infini se comportant comme un corps noir est divisé en tranches par des cavités infiniment profondes, espacées périodiquement et limitées par des parois planes parallèles. On porte le matériau solide à une température initiale uniforme et on le laisse ensuite se refroidir par conduction interne et par rayonnement. L'évolution de la température du solide avec ses cavités a été calculée numériquement. Des calculs similaires ont été conduits afin d'obtenir la vitesse de refroidissement du même matériau sans cavités. On a comparé les températures apparentes de brillance mesurées à grande distance à l'aide d'un radiomètre avec une résolution insuffisante pour distinguer entre une surface lisse et une surface comportant des cavités. On montre que dans une gamme étendue d'angles solides, le solide avec cavités peut avoir une température apparente plus élevée même lorsque les surfaces supérieures des tranches sont plus froides que la surface d'un solide homogène. Des phénomènes tels que celui qui vient d'être décrit peuvent être utiles pour interpréter les mesures du rayonnement infrarouge à partir de la surface lunaire.

Zusammenfassung—Ein fester, halbunendlicher schwarzer Körper ist von gleichmässig angeordneten unendlich tiefen Hohlräumen, die von ebenen parallelen Wänden begrenzt werden, in Streifen unterteilt. Der Festkörper wird auf gleichmässige Anfangstemperaturen gebracht und kühlt sich anschliessend durch Leitung im Inneren und Strahlung an den Oberflächen ab. Der Temperaturverlauf des Körpers mit Hohlräumen wurde numerisch berechnet. Ähnliche Rechnungen werden durchgeführt, um die Abkühlungsgeschwindigkeit des gleichen Materials ohne Hohlräume festzustellen. Ein Vergleich für die scheinbare Helligkeirstemperatur in grosser Entfernung wurde angestellt, wobei ein Temperaturstrahlungsdetektor verwendet wurde, dessen Auflösungsvermögen für eine Unterscheidung zwischen glatter Oberfläche und einen mit Hohlräumen nicht ausreichte. Es wird gezeigt, dass in einem weiten Bereich von Einfallswinkeln der Körper mit Hohlräumen eine höhere scheinbare Temperatur jaben kann, selbst mit kälteren Oberflächentemperaturen der Streifen als die Oberfläche des homogenen Festkörpers. Phänomene wie das hier beschriebene können bei der Auswertung von Messungen der Infrarotstrahlung von der Mondoberfläche von Nutzen sein.

Аннотация—Полуограниченное твердое черное тело разделено на плиты периодически чередующимися бесконечно глубокими полостями, ограниченными плоскими параллельными стенками. Твердый материал нагревается до однородной начальной температуры, а потом остывает в результате проводимости внутри материала и радиации на его поверхностях. Проведен численный расчет изменения температуры твердого тела с полостями, а также аналогичные расчеты с целью определения скорости охлаждения того же материала без полостей. Выполнено сравнение кажущихся яркостных температур, измеренных на большом расстоянии с помощью детектора теплового излучения, разрешающая способность которого недостаточно велика, чтобы различать гладкую поверхность от поверхности с полостями. Показано, что в большом диапазоне углов обзора твердое тело с полостями может иметь более высокие кажущиеся температуры, даже когда верхние поверхности плит холоднее поверхности однородного твердого тела. Явления, подобные описанным, используются для объяснения измерений инфракрасного излучения с поверхности луны.